

### Follow-up Discussion & Activities

- Primitive Pythagorean triples are Pythagorean triples that are not multiples of smaller triples. The first five primitive triples are:  
3, 4, 5    5, 12, 13    8, 15, 17    7, 24, 25    20, 21, 29  
Explain the concept of primitive Pythagorean triples to students and see if they can discover the first five primitive triples. Remind students that knowledge of a few primitive Pythagorean triples can help them create a large list of triples since any multiple of a Pythagorean triple is also a triple.
- Maps can provide excellent practice with coordinate grids and the distance formula. Give students a map with a coordinate grid overlay and have them use the distance formula to calculate the distances between various places on the map, keeping the map scale in mind as they perform their calculations. Challenge students by providing a map with a four-quadrant coordinate plane overlay.
- Incorporate the Pythagorean Theorem and the distance formula into a discussion of latitude and longitude by using the activity found at [www.ed.arizona.edu/ward/Latitude/pythag.html](http://www.ed.arizona.edu/ward/Latitude/pythag.html). Students can use the latitudes and longitudes of the various NASA sites given, or they can find latitudes and longitudes of local or regional landmarks to use in their calculations.
- Have students seek real-world applications of the Pythagorean Theorem and the distance formula by conducting research and generating a list of careers that would require an understanding of these mathematical concepts.
- Trigonometry** is the branch of mathematics involving triangles and angles. If enough information is known about the angles and sides of a right triangle, certain trigonometric ratios can be used to find the measures of the unknown angles and sides. Introduce students to the three common trigonometric ratios (sine, cosine, tangent) and have them use these ratios to find the missing angles and sides of various right triangles. Show students the mnemonic device SOHCAHTOA, which can be used to remember the three common trigonometric ratios:

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}} \quad \cos A = \frac{\text{adjacent}}{\text{hypotenuse}} \quad \tan A = \frac{\text{opposite}}{\text{adjacent}}$$

### Suggested Internet Resources

Periodically, Internet Resources are updated on our web site at [www.LibraryVideo.com](http://www.LibraryVideo.com).

- [www.regentsprep.org/Regents/math/ALGEBRA/math-ALGEBRA.htm](http://www.regentsprep.org/Regents/math/ALGEBRA/math-ALGEBRA.htm)  
This web site from the Oswego City School District offers links to lessons, practice questions for students, and teacher resources about the Pythagorean Theorem and trigonometric ratios.
- [www.shodor.org/interactivate/activities/SquaringTheTriangle](http://www.shodor.org/interactivate/activities/SquaringTheTriangle)  
The Shodor Education Foundation presents an online interactivity in which students explore the reasoning behind the Pythagorean Theorem.
- [www-groups.dcs.st-and.ac.uk/~history/Biographies/Pythagoras.html](http://www-groups.dcs.st-and.ac.uk/~history/Biographies/Pythagoras.html)  
This web site contains an informative biography of Pythagoras and links to discussions of various topics in the history of mathematics.

### Suggested Print Resources

- Cuomo, Serafina. *Ancient Mathematics*. Routledge, New York, NY; 2001.
- Derbyshire, John. *Unknown Quantity: A Real and Imaginary History of Algebra*. National Academies Press, Washington, D.C.; 2006.
- Hawking, Stephen. *God Created the Integers: The Mathematical Breakthroughs That Changed History*. Running Press Book Publishers, Philadelphia, PA; 2005.
- Maor, Eli. *Trigonometric Delights*. Princeton University Press, Princeton, NJ; 2002.

### TEACHER'S GUIDE

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# Algebra

## for Students™

### The Pythagorean Theorem & Right Triangles

Grades 7-12

In algebra, students are challenged to make a leap, from the concrete world of numbers and real objects, to an abstract one of letters and symbols. *Algebra for Students* is designed to help students to become more comfortable in the abstract world of algebra through the exploration of problems in the real world, from using a system of linear equations to calculate the cost of a sushi roll to using a quadratic function to describe the path of a kicked football. Animated graphics, real-life locales and vibrant young hosts help to explain math concepts, highlight multiple ways of approaching a problem, illustrate common pitfalls to avoid and tackle some typical test questions.

This guide provides a program overview, background knowledge needed for understanding, vocabulary, discussion questions and activities, tips for using a calculator, as well as print and Internet resources to supplement the teaching of targeted algebra concepts.



## Program Overview

Though Greek mathematician Pythagoras of Samos is credited with discovering the famous theorem that bears his name, historical evidence reveals that the mathematics contained in the Pythagorean Theorem was used by the ancient Egyptians, several thousand years before the birth of Pythagoras. The Pythagorean Theorem states that the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse, which is represented by the formula  $a^2 + b^2 = c^2$ . A set of three integers that satisfies the formula for the Pythagorean Theorem is known as a Pythagorean triple.

The Pythagorean Theorem has applications to many aspects of everyday life. In construction, it helps build sturdy walls by making sure corners are right angles and confirming the lengths of diagonal cross braces. The power of the Pythagorean Theorem is that it can be used in any situation involving a right triangle — if you know the lengths of two sides of the triangle, you can use the theorem to find the length of the third side.

Another important application of the Pythagorean Theorem is its use in the distance formula. The distance formula is used to calculate the distance between two points on the coordinate plane, and is derived from the Pythagorean Theorem — the two points are two vertices of a right triangle, and the distance between the points is the length of the hypotenuse of the triangle. When the distance between two locations cannot be measured easily, the locations can be converted into points on a coordinate grid, thus allowing the use of the distance formula.

## Background

Before studying the content discussed in the video, students should already be able to:

- Use the rules of exponents to perform calculations involving exponents.
- Perform calculations involving square roots.
- Identify the parts of the Cartesian coordinate system and plot points within it.
- Manipulate equations to isolate a variable on one side of the equation.

## Vocabulary

**right angle** — An angle that measures exactly  $90^\circ$ .

**right triangle** — A triangle that has one  $90^\circ$ , or right, angle.

**leg** — One of the two sides of a right triangle that forms the right angle.

**hypotenuse** — The longest side of a right triangle. The hypotenuse is opposite the right angle.

**Pythagorean Theorem** — A theorem stating that the sum of the squares of the legs of a right triangle is equal to the square of the hypotenuse. The formula for the Pythagorean Theorem is  $a^2 + b^2 = c^2$ .

**Pythagorean triple** — A set of three integers that satisfies the formula for the Pythagorean Theorem,  $a^2 + b^2 = c^2$ . (Continued)

**square root** — A factor of a number that when squared gives the number. The principal square root of a number  $x$  is  $\sqrt{x}$ , and the negative square root of a number  $x$  is  $-\sqrt{x}$ .

**distance formula** — A formula used to calculate the distance between two points on the coordinate plane. For two points  $(x_1, y_1)$  and  $(x_2, y_2)$ , the formula for the distance ( $d$ ) between the points is  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ .

## Pre-viewing Discussion

- Remind students that a perfect square number has an integer principal square root, and that it is a good idea to know the first twelve perfect squares and their corresponding square roots. Give students a set of objects (buttons, plastic discs, etc.) and encourage them to explore patterns with perfect squares. Can students find a way to use the objects to demonstrate why numbers like 16, 49 and 81 are perfect squares?
- Have students explore their homes or local environments to uncover some triangles in the real world. They can sketch their discoveries and describe their locations. In class, look at these triangles together and determine which ones can be classified as right triangles. Have students explain how they made their classifications.
- Reinforce plotting on the Cartesian coordinate system by having students create word puzzles using four-quadrant coordinate grids. Students should plot 26 points on their grids and label each point with a different letter of the alphabet. Then, they can create an encoded word puzzle (person, place, title, etc.) using the coordinates in place of the letters. Have students switch papers and solve their classmates' puzzles.

## Using a Calculator

The calculator is an invaluable tool when studying the Pythagorean Theorem, the distance formula, and trigonometric ratios. The following keys are very helpful:

- The  $(x^2)$  key allows you to find the square of a number.
- The  $(\sqrt{\quad})$  key allows you to find the square root of a number.
- The  $(\text{SIN})$ ,  $(\text{COS})$  and  $(\text{TAN})$  keys allow you to find the value of the appropriate trigonometric ratio for a given angle. When using trigonometric ratios, make sure that the calculator is in the proper mode (while many problems use degrees, there are situations where the angles for the ratios are expressed in radians).

Different calculators sometimes require different keys or key strokes to perform an operation. Sometimes the primary function of a key on one calculator appears as the secondary function of a key on another calculator. Encourage students to practice performing different functions on their calculators. Getting to know how their own calculator works is an important part of being a savvy algebra student.

## Problems

1. A giant slide has a height of 30 feet and occupies 50 feet of space on the ground. Find the length of the slide. Round your answer to the nearest tenth.
2. A football field has a length of 120 yards and a diagonal distance of 131.3 yards. Find the width of the football field. Round your answer to the nearest tenth.
3. When a coordinate grid is placed over the map of a township, the high school is located at (3,2), while the local college is located at (-2,5). If the scale of the grid is in miles, then how far apart are the high school and the college? Round your answer to the nearest mile.

## Solutions

1. The length of the slide represents the hypotenuse ( $c$ ) of a right triangle, and the other two dimensions represent the legs ( $a$  and  $b$ ) of the triangle:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (30)^2 + (50)^2 &= c^2 \\ 900 + 2500 &= c^2 \\ 3400 &= c^2 \\ \sqrt{3400} &= c \\ 58.3095 &\approx c \end{aligned}$$

The length of the slide is 58.3 feet.

2. The length and width of the football field represent the legs ( $a$  and  $b$ ) of a right triangle, and the diagonal is the hypotenuse ( $c$ ) of the triangle:

$$\begin{aligned} a^2 + b^2 &= c^2 \\ (120)^2 + b^2 &= (131.3)^2 \\ 14400 + b^2 &= 17239.69 \\ -14400 &\quad -14400 \\ b^2 &= 2839.69 \\ b &= \sqrt{2839.69} \\ b &\approx 53.2887 \end{aligned}$$

The width of the football field is 53.3 yards.

3. To find the distance, substitute the coordinates for the high school and the college into the distance formula and simplify:

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ d &= \sqrt{(-2 - 3)^2 + (5 - 2)^2} \\ d &= \sqrt{(-5)^2 + (3)^2} \\ d &= \sqrt{25 + 9} \\ d &= \sqrt{34} \\ d &\approx 5.83095 \end{aligned}$$

The high school and the college are approximately 6 miles apart.