- Show students the difference between the number of terms in a polynomial and the degree of a polynomial. While the polynomial $3x^2 + 7$ has two terms, it is a second degree polynomial because of the $x^2$ in the term $3x^2$. The polynomial $2x^4 + 5$ also has two terms, but it is a fourth degree polynomial because of the $x^4$ in the term $2x^4$.
- Some polynomials fit special patterns that make factoring much simpler. Have students create flash cards listing the different methods of factoring polynomials, including special factoring methods like the difference of squares and perfect square trinomials, and examples of polynomials factored using each method. Students can keep the flash cards in their notebook or folder for easy reference.
- Students who are having trouble multiplying polynomials using either FOIL or the distributive property may find it easier to multiply vertically. Show students how two polynomial expressions, like $2x + 5$ and $3x - 4$, can be multiplied in a similar way as they would vertically multiply numbers:

$$\begin{array}{r} 2x + 5 \\ \times \quad 3x - 4 \\ \hline -8x - 20 \\ 6x^2 + 15x \quad \\ \hline 6x^2 + 7x - 20 \end{array}$$

- Have students conduct research and create a list of careers that would require an understanding of how to solve polynomial equations. Explain or provide examples of how this mathematical concept is applied to each field listed. How might this factor into the work of an environmentalist, an engineer or a doctor?

## Suggested Internet Resources

Periodically, Internet Resources are updated on our web site at www.LibraryVideo.com.

- **www.pen.k12.va.us/Div/Winchester/jhhs/math/lessons/algebra.html**
  The Mathematics Department of John Handley High School in Winchester, VA has assembled numerous algebra ancillary materials, including several puzzles and worksheets on factoring polynomials.
- **illuminations.nctm.org/WebLinks.aspx**
  This National Council of Teachers of Mathematics web site offers over 700 website links organized by standard.

## Suggested Print Resources


- Derbyshire, John. *Unknown Quantity: A Real and Imaginary History of Algebra*. National Academies Press, Washington, D.C.; 2006.


***(Continued)***






- Kaplan, Robert and Ellen Kaplan. *The Nothing That Is: A Natural History of Zero*. Oxford University Press, New York, NY; 2000.
- Muschla, Judith A. and Gary Robert Muschla. *Algebra Teacher's Activities Kit: 150 Ready-to-Use Activities with Real-World Applications*. John Wiley & Sons, Incorporated, Hoboken, NJ; 2003.


## Using a Calculator

If students are using the graphing method to solve a polynomial equation, the graphing calculator is a useful tool and an alternative to paper-and-pencil graphing. The following keys are very helpful in finding these coordinates:

- The (TRACE) key allows you to move a cursor along the graph of an equation and locate the *x*-intercepts. The *x*- and *y*- coordinates that express the location of the cursor on the graph appear at the bottom of the screen.
- The (TABLE) feature organizes the data from the graph into a table.
- The (CALC) menu, found on many graphing calculators, has an option that will let students find the coordinates for the *x*-intercepts of the equation.

Try graphing different polynomial equations, especially those with polynomial expressions that are not factored easily.

Different calculators sometimes require different keys or key strokes to perform an operation. Encourage students to practice performing different functions on their calculators. Getting to know how their own calculator works is an important part of being a savvy algebra student.

**TEACHER'S GUIDE**


**James Fanelli, M.A. & Megan Carnate, M.Ed.**
Curriculum Specialists, Schlessinger Media


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P.O. Box 580, Wynnewood, PA 19096 • 800-843-3620
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# Algebra for Students™

## Polynomials

### Grades 7-12

In algebra, students are challenged to make a leap, from the concrete world of numbers and real objects, to an abstract one of letters and symbols. *Algebra for Students* is designed to help students to become more comfortable in the abstract world of algebra through the exploration of problems in the real world, from using a system of linear equations to calculate the cost of a sushi roll to using a quadratic function to describe the path of a kicked football. Animated graphics, real-life locales and vibrant young hosts help to explain math concepts, highlight multiple ways of approaching a problem, illustrate common pitfalls to avoid and tackle some typical test questions.

This guide provides a program overview, background knowledge needed for understanding, vocabulary, discussion questions and activities, tips for using a calculator, as well as print and Internet resources to supplement the teaching of targeted algebra concepts.

## Program Overview

Polynomial expressions are used to model many real-life situations, like the volume of water needed for a swimming pool or the area of a walkway surrounding a pool. All of the arithmetic operations applied to real numbers can be used with polynomials. The distributive property and the FOIL method are valuable tools when multiplying polynomials.

When solving a polynomial equation, factoring is often used to simplify a polynomial expression. For example, many quadratic trinomials can be expressed as the product of two binomials. Some polynomials fit special patterns, like the difference of squares, which make factoring much simpler. Once factored, a polynomial equation can be solved using the zero product property.

Graphing is a useful method of solving some polynomial equations, especially those with polynomial expressions that are not factored easily. The solutions of a polynomial equation, known as its roots or zeros, are represented by the $x$-intercepts of the graph. Polynomial equations whose solutions are not integers can be graphed on a graphing calculator, which has options that enable accurate approximations of the solutions.

## Background

Before studying the content discussed in the video, students should already be able to:

- Apply and use the Distributive Property.
- Find the greatest common factor (GCF) of a set of monomials.
- Explain the meaning of "like terms" and how to combine like terms.
- Distinguish between graphs and written equations of linear and nonlinear functions.

## Vocabulary

**monomial —** A number, a variable, or a product of a number and one or more variables.

**constant —** A monomial that is a real number; or a known quantity that stays the same.

**polynomial —** A monomial or a sum or difference of monomials.

**binomial —** A polynomial with two terms.

**trinomial —** A polynomial with three terms.

**distributive property —** A rule stating that the product of a number and the sum or difference of two numbers equals the product of the sum or difference of the two numbers. For any real numbers $a$, $b$ and $c$, $a(b + c) = ab + ac$ and $a(b - c) = ab - ac$.

**FOIL method —** A method for multiplying two binomials by finding the sum of the products of the *F*irst terms, the *O*utside terms, the *I*nside terms and the *L*ast terms. ***(Continued)***



**factoring —** Rewriting a polynomial expression as a product. For example, the polynomial expression $2x^2 + 5x - 12$ factors into the product $(2x - 3)(x + 4)$.

**zero product property —** A rule stating that the product of any number and zero is zero. For any real numbers $a$ and $b$, if $ab = 0$, then $a = 0$ or $b = 0$.

**quadratic trinomial —** A polynomial with three terms that has the form $ax^2 + bx + c$.

**perfect square —** A number that can be expressed as the product of two identical integers. For example, the number 64 is a perfect square since $8 \cdot 8 = 64$. A polynomial is a perfect square if it can be expressed as the product of two identical polynomial expressions. The polynomial $4x^2$ is a perfect square since $2x \cdot 2x = 4x^2$.

**difference of squares —** A polynomial expression containing one perfect square that is subtracted from another perfect square. For any real numbers a and b, the difference of squares $a^2 - b^2$ has the factors $(a + b)(a - b)$.

**roots (or zeros) —** The $x$-intercepts of a quadratic function; or the solutions of a quadratic equation.

***x*-intercept —** The $x$-value in an ordered pair describing the point at which a graph crosses the $x$-axis. For example, since the graph of the function $y = 2x - 4$ intersects the $x$-axis at (2, 0), the $x$-intercept is 2.

## Pre-viewing Discussion

- A basic understanding of the operations involving monomials is an essential skill for solving problems involving polynomials. Write a pair of integers (like 6 and 2) and a pair of monomials (like $6x$ and $2y$) on the board and have students compare and contrast the addition, subtraction, multiplication and division of each pair. How are operations involving monomials similar to operations involving integers? How are they different?
- Knowing how to find the greatest common factor, or GCF, of a set of monomials will help students more easily factor polynomial expressions. Have students practice finding the GCF of sets of numbers, like 20 and 36, and sets of monomials, like $8x^2$ and $2x$.

## Problems

A sandbox currently found on a playground is square. A proposed new sandbox will have a length 5 feet longer and a width 1 foot shorter than the current sandbox.

1. Write expressions for the length and width of the new sandbox.
2. Write a polynomial expression for the area of the new sandbox.
3. Suppose the new sandbox has a maximum allotted area of 55 ft². Find the length and width of the new sandbox.



## Solutions

1. Let $x$ equal the length of one side of the current sandbox.
   New length = $x + 5$ (5 feet longer than the current length)
   New width = $x - 1$ (1 foot shorter than the current width)
2. The new sandbox area can be calculated using $A = lw$, the formula for the area of a rectangle:
   $A = lw$
   $A = (x + 5)(x - 1)$
   $A = x^2 + 5x - x - 5$
   $A = x^2 + 4x - 5$
   The polynomial expression representing the area of the new sandbox is $x^2 + 4x - 5$.
3. The area of the new sandbox, represented by the polynomial expression $x^2 + 4x - 5$, equals 55 ft², which means $x^2 + 4x - 5 = 55$. Now solve for $x$:
   $x^2 + 4x - 5 = 55$
   $x^2 + 4x - 5 = 55$ Subtract 55 from each side.
   $x^2 + 4x - 60 = 0$ Factor the trinomial on the left side.
   $(x + 10)(x - 6) = 0$
   $x + 10 = 0$ or $x - 6 = 0$
   $x = -10$ or $x = 6$
   The distance cannot be negative, so $x = 6$. Substitute 6 for $x$ in the expressions for the new length and width of the sandbox:
   New length = $x + 5 = 6 + 5 = 11$ New width = $x - 1 = 6 - 1 = 5$
   The new sandbox will have a length of 11 feet and a width of 5 feet.

## Follow-up Discussion & Activities

- An essential skill in factoring is the ability to quickly determine the factors of numbers. Students should be able to recognize **prime numbers** — whole numbers greater than 1 whose only factors are 1 and the number itself, including the first few prime numbers: 2, 3, 5, 7, 11, 13, 17, and 19. Students should be able to find the prime factorization of **composite numbers** — whole numbers greater than 1 that have more than two factors. Utah State's National Library of Virtual Manipulatives, located at **nlvm.usu.edu/en/nav/topic_t_2.html**, has a "Factor Tree" activity for students who want extra factoring practice.
- Is it a polynomial? Emphasize to students that a polynomial does not contain any terms with variables in the denominator or any terms with variables under a radical sign. For example, $2x^3 + 5x + 7$ is polynomial, but $t - \frac{1}{t^2}$ is not. Write several expressions on the board and ask students to determine if each is a polynomial. If it is, classify it as a monomial, binomial or trinomial. If it is not a polynomial, have students explain why. ***(Continued)***