- Formulas are used in many aspects of everyday life. Have students research a specific profession, like an architect or a chemist, and create a poster illustrating some of the formulas used by these professions.
- Show students the formula for calculating simple interest, $I = prt$ (where I is the interest, p is the principal, r is the annual interest rate and t is the time in years). Discuss the differences between compound interest and simple interest with students. Given appropriate values, have students compare the interest earned on an amount of money via simple interest with the interest earned on the same amount via compound interest.
- In addition to the graphing calculator, spreadsheet programs are an effective way to enter and evaluate simple as well as complex formulas, like the one for calculating heat index, to investigate problems. Give students a complex formula and have them use a spreadsheet program to generate data and make graphs based on the formula. Remind students that spreadsheet programs use column letters and row numbers to describe the location of cells. Furthermore, these letter-number combinations serve as variables in spreadsheet formulas.

Using a Calculator

You can use a graphing calculator to explore patterns and the (ANS) key is a helpful tool for doing so. (ANS) refers to the last answer the calculator got. This is useful in many situations, particularly when repetition plays a role. For example, suppose you start with $35 and quadruple it every year. How much money do you have in the years that follow? Simply follow the steps below:

1. Enter 35 and then press the (ENTER) key. The calculator should come up with the answer 35.
2. To quadruple the answer, enter 4, press (ANS) (which refers to your previous answer of 35), and then press the (ENTER) key. The calculator should provide you with an answer of 140. You would have $140 the following year.
3. Now, press the (ENTER) key a few more times. It should reveal the amount of money you have in subsequent years (560, 2240, 8960,…). Each press of the (ENTER) key will evaluate the next term in the sequence.

Sometimes, there is more than one way to use a calculator to solve a problem. In addition, different calculators often require different keys or key strokes to perform an operation. Sometimes the primary function of a key on one calculator appears as the secondary function of a key on another calculator. Encourage students to practice performing different operations on their calculators. Getting to know how their own calculator works is an important part of being a savvy algebra student.

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Suggested Internet Resources

Periodically, Internet Resources are updated on our web site at www.LibraryVideo.com.

- **jwilson.coe.uga.edu/EMAT6680/Parveen/Fib_nature.htm**
  This web site from the University of Georgia illustrates many examples of Fibonacci numbers in nature, including plants, animals and even humans.
- **www.wtamu.edu/academic/anns/mps/math/mathlab/beg_algebra/beg_alg_tut35_reason.htm**
  This tutorial from West Texas A&M University explores the inductive reasoning skills used when analyzing patterns and sequences.
- **nlvm.usu.edu/en/nav/topic_t_2.html**
  The interactive library at Utah State's National Library of Virtual Manipulatives has activities on patterns and inductive reasoning that are connected to NCTM standards.

Suggested Print Resources

- Berlinski, David. *Infinite Ascent: A Short History of Mathematics*. Random House Publishing Group, New York, NY; 2005.
- Bodanis, David. *E=mc2: A Biography of the World's Most Famous Equation*. Penguin Putnam Incorporated, New York, NY; 2001.
- Livio, Mario. *The Golden Ratio: The Story of Phi, the World's Most Astonishing Number*. Broadway Books, New York, NY; 2002.
- Posamentier, Alfred S. and Ingmar Lehmann. *Pi: A Biography of the World's Most Mysterious Number*. Prometheus Books, Amherst, NY; 2004.

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TEACHER'S GUIDE

Algebra for Students™

Patterns & Formulas

Grades 7-12

In algebra, students are challenged to make a leap, from the concrete world of numbers and real objects, to an abstract one of letters and symbols. *Algebra for Students* is designed to help students to become more comfortable in the abstract world of algebra through the exploration of problems in the real world, from using a system of linear equations to calculate the cost of a sushi roll to using a quadratic function to describe the path of a kicked football. Animated graphics, real-life locales and vibrant young hosts help to explain math concepts, highlight multiple ways of approaching a problem, illustrate common pitfalls to avoid and tackle some typical test questions.

This guide provides a program overview, background knowledge needed for understanding, vocabulary, discussion questions and activities, tips for using a calculator, as well as print and Internet resources to supplement the teaching of targeted algebra concepts.

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Program Overview

Patterns are predictable sequences or designs and can be found in many aspects of everyday life. We use patterns to predict events and to make information more easily accessible: knowing the layout of the supermarket makes it easier to find the items, knowing your daily schedule makes it easier to budget time and move from one task to the next, and so on. Information that follows a recognizable numerical pattern can be converted into a formula — a mathematical equation representing a rule that explains the relationship between information.

Formulas are functions derived by analyzing data that is collected from observable patterns. Mathematicians and scientists frequently use formulas, and they often make graphs and tables to illustrate the information represented by the formula. One interesting property of a function in two variables is that its inverse is obtained when the x-values and y-values are switched — that is, when the coordinates of its ordered pairs are reversed. The graph of a function and its inverse are reflections over the line $y = x$.

The beauty of many formulas — like the formula for calculating compound interest — is that they have already been developed and proven accurate by someone else, thus allowing us to feel confident when we use them. Sometimes we see only the end result of using a complex formula. Meteorologists collect all sorts of weather data and input that information into sophisticated mathematical models and formulas to create weather forecasts.

Background

Before studying the content discussed in the video, students should already be able to:

- Apply basic arithmetic operations and processes.
- Evaluate algebraic expressions.
- Inductively reason with number patterns.
- Graph in Quadrant I of the coordinate plane.

Vocabulary

pattern — A predictable sequence or design. For example, a checkerboard has a predictable pattern of red and black squares, while the number sequence "2, 4, 8, 16, 32, …" is a pattern because each successive term is double the previous term.

formula — A mathematical equation that represents the rule explaining the relationship between certain quantities. For example, the volume of a rectangular prism is related to its length, width and height, and is represented by the formula $A = lwh$.

(Continued)

inverse — For a function, the inverse is obtained by switching its x-values and y-values (the coordinates of the ordered pairs). For example, a function containing the ordered pairs (1,5), (3,7), and (5,9), has an inverse function containing the ordered pairs (5,1), (7,3), and (9,5). One property of the graph of a function and its inverse is that they are reflections over the line $y = x$.

common difference — The difference between terms in a sequence. For example, in the sequence "3, 14, 25, 36, 47, …," the common difference is +11.

compound interest — Interest earned on both the principal and previous interest payments. The formula for compound interest is $A=P(1 + \frac{r}{n})^{nt}$, where A represents the total amount of money, P is the principal (the initial amount of money), r is the yearly interest rate (expressed in decimal form), n is the number of times the interest is compounded per year (typically expressed in a phrase like "compounded quarterly," which indicates that interest is compounded 4 times per year) and t is the number of years the principal is gaining interest.

Pre-viewing Discussion

- Have students examine their own lives and their local environment for various patterns. Students can share their findings with classmates, including a description of the influence of these patterns on themselves and the world around them.
- Discuss the use of inductive and logical reasoning in problem solving. Have students solve word and logic problems to increase their familiarity with inductive and logical reasoning.
- Discuss the importance of units in calculations. Introduce the idea of keeping track of units when solving a problem, known as **dimensional analysis**, and show students the value of keeping track of units while using a formula.

Problems

1. Write an equation that describes the relationship shown in the table.

x	y
1	4
2	7
3	10
4	13
5	16

2. Jonathan invests $750 in a savings account with an interest rate of 5% compounded every two weeks. How much money will Jonathan have after 10 years?

Solutions

1. The pattern of the data suggests that as x increases by 1, y increases by 3. The difference in y-values is three times the difference in x-values, which seems to indicate that $y = 3x$. Check this solution using a few values of x:

   If $x = 1$, then $y = 3(1)$, or 3 — but the y-value for $x = 1$ is 4, not 3.
   If $x = 2$, then $y = 3(2)$, or 6 — but the y-value for $x = 2$ is 7, not 6.

   The equation $y = 3x$ is <u>incorrect</u>. However, examination of this new data reveals that each value of x substituted into $y = 3x$ yields a value of y that is 1 away from the original data. This pattern suggests that each y-value equals three times the corresponding x-value <u>plus 1</u>. Algebraically, this means $y = 3x + 1$.

   Check this revised solution using a few values of x:

   $x = 1$: $y = 3(1) + 1$, $y = 3 + 1$, $y = 4$ ✓
   $x = 4$: $y = 3(4) + 1$, $y = 12 + 1$, $y = 13$ ✓

2. To calculate the amount of money in Jonathan's savings account, substitute the values for the principal ($P = 750$), the interest rate ($r = 0.05$), the number of times the interest is compounded per year ($n = 26$) and the length of the investment ($t = 10$) into the compound interest formula and simplify:

   $A = P(1 + \frac{r}{n})^{nt}$
   $A = 750(1 + \frac{0.05}{26})^{26(10)}$
   $A = 750(1.001923077)^{260}$
   $A \approx 750(1.64793)$
   $A \approx 1235.95$

   Jonathan will have $1,235.95 after 10 years.

Follow-up Discussion & Activities

- Discuss special kinds of sequences, including arithmetic and geometric sequences. Remind students that some sequences are neither arithmetic nor geometric and may or may not have specific names. Give students several sequences and have them find the pattern, determine the kind of sequence (if applicable) and present their findings to the class.
- Explore number patterns by introducing students to the Fibonacci sequence, discovered by Italian mathematician Leonardo Fibonacci. Write the beginning of the sequence, "1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …," on the board and have students identify the rule and predict the next numbers in the sequence. Mathematicians and scientists have linked Fibonacci numbers to numerous different patterns in nature, including flower petals and architectural designs. Have students use print and electronic resources to find and report on objects that use Fibonacci numbers.

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