Follow-up Discussion & Activities

- Parabolas can be found in many aspects of the real world, including architecture, sports and business. Have students explore their surroundings for real-life examples or models of parabolas and conduct some research to confirm their findings. Students can share their discoveries. Was there anything that appeared to be a parabola but turned out not to be one?
- Quadratic equations can be solved by various methods, including making a table of values, graphing, using the quadratic formula, and **completing the square**. Have students practice solving quadratic equations by completing the square, a method that involves making the expression on one side of a quadratic equation a perfect square. Visit **www.algebralab.org/lessons/lesson.aspx?file=Algebra_completingthesquare.xml** for sample problems and helpful hints.
- The discriminant of the quadratic formula is $b^2 - 4ac$, the expression under the square root sign. Working in small groups, give students several quadratic equations and have them calculate the values of the discriminants and find the connection between the value of the discriminant and the number of roots for a quadratic equation (positive discriminant = 2 real roots; negative discriminant = 0 real roots; discriminant of zero = 1 real root).
- Introduce the concept of imaginary numbers by having students use the quadratic formula to solve a quadratic equation like $2x^2 + 5x + 8 = 0$. Once students have simplified the roots of the equation to $x = \frac{-5 \pm \sqrt{-39}}{4}$, explain that the square root of a negative number is known as an imaginary number and does not represent any number on the standard coordinate plane. Adventurous students can enhance their understanding of this topic by exploring the number i, equal to $\sqrt{-1}$, and how it can be used to solve the equation.

Suggested Internet Resources

Periodically, Internet Resources are updated on our web site at www.LibraryVideo.com.

- **www.nsa.gov/teachers/teach00006.cfm**
  The Mathematics Education Partnership Program offers learning units for all areas of math, including the "Kangaroo Conundrum: A Study Of A Quadratic Function" activity in the High School Algebra section.
- **www.ccsn.nevada.edu/math/qf_calc.htm**
  This web site from the Community College of Southern Nevada offers an easy-to-use Quadratic Formula Calculator.

Suggested Print Resources

- Derbyshire, John. *Unknown Quantity: A Real and Imaginary History of Algebra*. National Academies Press, Washington, D.C.; 2006. ***(Cont'd.)***

TEACHER'S GUIDE

- The Math Forum. *Dr. Math Explains Algebra: Learning Algebra Is Easy! Just Ask Dr. Math!* John Wiley & Sons, Incorporated, Hoboken, NJ; 2003.

Using a Calculator

The graphing calculator is a useful tool for finding the roots, or zeroes, of a function because you can do this in more than one way. The following keys/features are very helpful in finding these coordinates:

- The (TRACE) key allows you to move a cursor along the graph of an equation and locate the x-intercepts. The x- and y- coordinates that express the location of the cursor on the graph appear at the bottom of the screen.
- The (TABLE) feature organizes the data from the graph into a table. Remember that the graph crosses the x-axis when the values for y change signs.
- The (CALC) feature, found on many graphing calculators, has an option that will let students find the coordinates for the x-intercepts of the equation.

When using a graphing calculator to solve an equation using the quadratic formula, be sure to use the parenthesis key(s). They typically look like this: (() and ()) or (( )). These keys help to group operations that should be executed first.

Different calculators sometimes require different keys or key strokes to perform an operation. Encourage students to practice performing different functions on their calculators. Getting to know how their own calculator works is an important part of being a savvy algebra student.

TEACHER'S GUIDE

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TEACHER'S GUIDE

Algebra for Students™

Quadratic Functions

Grades 7-12

In algebra, students are challenged to make a leap, from the concrete world of numbers and real objects, to an abstract one of letters and symbols. *Algebra for Students* is designed to help students to become more comfortable in the abstract world of algebra through the exploration of problems in the real world, from using a system of linear equations to calculate the cost of a sushi roll to using a quadratic function to describe the path of a kicked football. Animated graphics, real-life locales and vibrant young hosts help to explain math concepts, highlight multiple ways of approaching a problem, illustrate common pitfalls to avoid and tackle some typical test questions.

This guide provides a program overview, background knowledge needed for understanding, vocabulary, discussion questions and activities, tips for using a calculator, as well as print and Internet resources to supplement the teaching of targeted algebra concepts.

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Program Overview

Many real-life situations, like the path of a kicked football or the trajectory of a fireworks shell, are modeled by quadratic functions. The graph of a quadratic function is a parabola and does not have a constant rate of change. This differs from the graph of a linear function, which graphs to a straight line and has a constant rate of change.

All parabolas have a lowest point or highest point that is known as its vertex. The axis of symmetry of a parabola passes through the vertex and divides the parabola into two symmetric halves. The coordinates of the vertex of a parabola can be calculated by substituting the coefficients of the quadratic function of the parabola into certain formulas. The values of the coefficients and constant of a quadratic function affect the graph of the parabola of the function.

Quadratic equations can be solved by several methods, including making a table of values, graphing, factoring, and using the quadratic formula. While finding the solutions to some quadratic equations is often difficult using a table of values, graphing, or factoring, using the quadratic formula is a guaranteed way to find the solution to a quadratic equation, even when a quadratic equation has no solution.

Background

Before studying the content discussed in the video, students should already be able to:

- Perform mathematical operations involving polynomials.
- Explain function notation and how to compute values of functions.
- Identify the graphs and written equations of linear and nonlinear functions.

Vocabulary

linear function — A first degree polynomial function that can be expressed in the form $y = mx + b$. The graph of a linear function is a straight line and has a constant rate of change.

quadratic function — A second degree polynomial function that can be expressed in the form $y = ax^2 + bx + c$, where $a \neq 0$. The graph of a quadratic function is a parabola and does not have a constant rate of change.

parabola — A symmetric curve that is the graph of a quadratic function.

vertex — The highest or lowest point on a parabola. The coordinates (h, k) of the vertex of a parabola can be calculated using the quadratic function of the parabola and the formulas $h = -\frac{b}{2a}$ and $k = f(-\frac{b}{2a})$.

axis of symmetry — The vertical line that divides a parabola into two symmetric halves. The axis of symmetry of a parabola passes through the vertex of the parabola.

(Continued)

minimum — The lowest point on the graph of a parabola that opens upwards. For example, the vertex of the graph of the quadratic function $y = x^2$ is a minimum.

maximum — The highest point on the graph of a parabola that opens downwards. For example, the vertex of the graph of the quadratic function $y = -x^2$ is a maximum.

roots (or zeros) — The x-intercepts of a quadratic function; or the solutions of a quadratic equation.

quadratic equation — A quadratic function set equal to 0. For example, the quadratic function $y = 3x^2 + 2x + 5$ can be rewritten as the quadratic equation $3x^2 + 2x + 5 = 0$.

***x*-intercept —** The x-value in an ordered pair describing the point at which a graph crosses the x-axis. For example, since the graph of the function $y = 3x - 6$ intersects the x-axis at (2, 0), the x-intercept is 2.

quadratic formula — A formula used to calculate the solutions of a quadratic equation. The solutions of a quadratic equation of the form $ax^2 + bx + c = 0$ are given by the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Pre-viewing Discussion

- Give students graphs of a linear function and a quadratic function, but do not label them as such. Have students make a table of values based on the graph of each function, then use the information in the table to describe differences between the two graphs. Challenge students with a graph of the equation $x = y^2$. How is this graph like that of the quadratic function? Is it a function?
- The graphing calculator lends itself particularly well to exploring the graphs of quadratic functions. Review with students the basics of making a graph on the graphing calculator, including how to input the equation and adjusting the display window to get the best view of the graph. Since use of the quadratic formula requires taking the square root of a number, students should also be able to locate and use the square root button on their calculators.

Problems

1. Cynthia hits a golf ball that can be represented by the quadratic function $y = -0.0017x^2 + 0.255x$, where x is the horizontal distance the ball travels in yards, and y is the height of the ball, also in yards. Determine the horizontal distance Cynthia's golf ball traveled by making a table of values, with x increasing in increments of 25 yards.
2. Find the vertex (h, k) of the parabola represented by the quadratic function $y = 3x^2 - 12x + 8$ by using the quadratic function itself to determine the values of h and k.
3. Use the quadratic formula to solve the equation $9x^2 + 5x - 12 = 0$. Round your solutions to the nearest hundredth.

Solutions

1.

Horizontal Distance (yds)	Height in the Air (yds)
0	0
25	5.3125
50	8.5
75	9.5625
100	8.5
125	5.3125
150	0

Cynthia's golf ball traveled a horizontal distance of 150 yards.

2. The coordinates of the vertex of a quadratic function are designated (h, k). Using the appropriate coefficients from $y = 3x^2 - 12x + 8$, find h:

$$h = -\frac{b}{2a} = -\frac{-12}{2(3)} = \frac{12}{6} = 2$$

To find k, evaluate $y = 3x^2 - 12x + 8$ with 2, the value of $-\frac{b}{2a}$, substituted for x:

$$k = 3(2)^2 - 12(2) + 8$$
$$k = 3(4) - 24 + 8$$
$$k = 12 - 24 + 8$$
$$k = -4$$

The coordinates of the vertex are (2, -4).

3. Substitute the appropriate coefficients from $9x^2 + 5x - 12 = 0$ into the quadratic formula and simplify:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(9)(-12)}}{2(9)}$$

$$x = \frac{-5 \pm \sqrt{457}}{18}$$

$$x \approx \frac{-5 \pm 21.38}{18}$$

$$x \approx \frac{-5 + 21.38}{18} \quad \text{or} \quad x \approx \frac{-5 - 21.38}{18}$$

$$x \approx 0.91 \qquad x \approx -1.47$$