Using a Calculator

The (^) key is a quick and easy way to raise any number to any power. It is found on most scientific and graphing calculators. If you want to calculate 5 raised to the fourth power, simply enter (5), then press (^), enter (4), then press (=). You should get 625 as your answer. A few helpful things to remember when working with exponents on a scientific or graphing calculator:

- When raising a number to a fractional power, enclose the fraction in parentheses.
- When raising a negative base to a power, enclose the negative number in parentheses. For example, -3^4 is not the same as $(-3)^4$.

Different calculators sometimes require different keys or key strokes to perform an operation. Encourage students to practice performing different functions on their calculators. Getting to know how their own calculator works is an important part of being a savvy algebra student.

Follow-up Discussion & Activities

- The rules of exponents are numerous and potentially confusing for students. Have students create a chart that gives examples of the rules of exponents, including zero power, negative power, multiplication, division, power of a power, and power of a product. Students can keep the charts in their notebooks or folders for easy reference.
- Have students create posters to reinforce the usefulness of expressing numbers in scientific notation. Working in small groups, students can make posters highlighting the planets in the solar system, including the mass of each planet and its distance from the Sun. Instruct students to make one poster using numbers expressed in standard notation, and a second poster using numbers expressed in scientific notation.
- Explore the differences between exponential functions and quadratic functions by graphing the functions $y = x^2$ and $y = 2^x$ on the same first-quadrant graph. Have students compare the growth rate of an exponential function to that of the quadratic function, using their graphs to support their explanation.
- Radioactive elements, such as the carbon-14 and uranium-235, decay into atoms of other elements over time. **Half-life** is the amount of time it takes for half of the radioactive atoms in a sample to decay into other elements. Have students explore this topic by researching various radioactive element isotopes. Students can create a poster illustrating the location and uses of the radioactive isotope, and a graph showing how a given amount of the isotope exponentially decays over time.

- A geometric sequence is a sequence in which each term equals the previous term multiplied by a constant, often called the **common ratio**. Have students look for applications of a geometric sequence in real life, then display the growth or decay by a common ratio in a graph. Discuss the similarities between geometric sequences and exponential functions.

Suggested Internet Resources

Periodically, Internet Resources are updated on our web site at www.LibraryVideo.com.

- **www.wtamu.edu/academic/anns/mps/math/mathlab/beg_algebra/index.htm**
  These tutorials from West Texas A&M University offer step-by-step instructions on exponents, scientific notation, and a variety of other algebra topics.
- **www.nsa.gov/teachers/teach00006.cfm**
  The Mathematics Education Partnership Program offers learning units for all areas of math, including the "Making Money and Spreading the Flu!" activity in the High School Algebra section.

Suggested Print Resources

- Berlinski, David. *Infinite Ascent: A Short History of Mathematics*. Random House Publishing Group, New York, NY; 2005.
- Derbyshire, John. *Unknown Quantity: A Real and Imaginary History of Algebra*. National Academies Press, Washington, D.C.; 2006.
- Maor, Eli. *e: The Story of a Number*. Princeton University Press, Princeton, NJ; 1998.

TEACHER'S GUIDE

James Fanelli, M.A. & Megan Carnate, M.Ed.
Curriculum Specialists, Schlessinger Media

TITLES IN THIS SERIES

- ANALYZING INEQUALITIES
- EXPONENTIAL FUNCTIONS
- FUNCTIONS & RELATIONS
- LINEAR EQUATIONS & SLOPE
- PATTERNS & FORMULAS
- POLYNOMIALS
- PYTHAGOREAN THEOREM & RIGHT TRIANGLES
- QUADRATIC FUNCTIONS
- SYSTEMS OF LINEAR EQUATIONS
- VARIABLES, EXPRESSIONS & EQUATIONS

Teacher's Guide and Program Copyright 2007 by Schlessinger Media, a division of Library Video Company
P.O. Box 580, Wynnewood, PA 19096 • 800-843-3620
Executive Producer: Andrew Schlessinger
Programs produced and directed by Center City Film and Video

B0782
V6732

Algebra for Students™

Exponential Functions

Grades 7-12

In algebra, students are challenged to make a leap, from the concrete world of numbers and real objects, to an abstract one of letters and symbols. *Algebra for Students* is designed to help students to become more comfortable in the abstract world of algebra through the exploration of problems in the real world, from using a system of linear equations to calculate the cost of a sushi roll to using a quadratic function to describe the path of a kicked football. Animated graphics, real-life locales and vibrant young hosts help to explain math concepts, highlight multiple ways of approaching a problem, illustrate common pitfalls to avoid and tackle some typical test questions.

This guide provides a program overview, background knowledge needed for understanding, vocabulary, discussion questions and activities, tips for using a calculator, as well as print and Internet resources to supplement the teaching of targeted algebra concepts.

Program Overview

Exponents and exponential notation are shorthand ways of dealing with repeated multiplication. Numbers expressed in exponential notation have a base and an exponent. Like all numbers, there are rules of exponents that must be followed to ensure accurate calculations. Exponents are of vital importance to scientific notation — a concise way of writing very large or very small numbers.

An exponential function is a function in which the variable is an exponent. The graphs of exponential functions are curves and do not have a constant rate of change. The doubling of money over a period of time is represented by an exponential growth function. A comparison of the graphs of an exponential growth function and a linear growth function shows that linear functions grow steadily over time while exponential functions grow at a tremendous rate over time.

The successive bounces of a dropped rubber ball or the elimination of teams from a college basketball tournament are represented by exponential decay functions. While linear decay functions decrease steadily over time, exponential decay functions decrease at a tremendous rate over time.

Background

Before studying the content discussed in the video, students should already be able to:

- Apply some of the rules of exponents and scientific notation.
- Use the order of operations to solve problems.
- Graph ordered pairs in the coordinate plane.
- Recognize and graph linear functions.

Vocabulary

base — A factor that is multiplied repeatedly. In 5^4, the base is 5 since $5^4 = 5 \cdot 5 \cdot 5 \cdot 5$.

exponent — A number that indicates how many times a base is multiplied by itself. In 3^7, the exponent is 7 and indicates that the 3, the base, should be multiplied by itself seven times: $3^7 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$.

power — An expression that contains a base and an exponent. The expression 2^6 can be read as "the sixth power of 2" or "2 raised to the sixth power."

zero power — Any non-zero number raised to the zero power is equal to 1. For any non-zero real number x, $x^0 = 1$.

negative power — A number raised to a negative power is equal to 1 divided by the number raised to the positive power. For any non-zero real number x and any integer n, $x^{-n} = \frac{1}{x^n}$.

multiplying powers with like bases — When multiplying two or more powers that share the same base, add the exponents. For example, $3^4 + 3^5 = 3^{4+5} = 3^9$.

(Continued)

dividing powers with like bases — When dividing two or more powers that share the same base, subtract the exponents. For example, $5^7 - 5^4 = 5^{7-4} = 5^3$.

raising a power to a power — When raising a power to a power, multiply the exponents. For example, $(4^3)^2 = 4^{3 \cdot 2} = 4^6$.

power of a product — When raising a product to a power, distribute the power to each factor and simplify. For example, $(4x)^3 = 4^3x^3 = 64x^3$.

order of operations — The rules telling what order to perform operations within an equation: (1) simplify within parentheses or other brackets; (2) simplify exponents; (3) multiply and divide from left to right; and (4) add and subtract from left to right.

scientific notation — A way of writing a number as the product of a power of 10 and a decimal number greater than or equal to 1 and 10. In scientific notation, the number 0.0000000082 is written 8.2×10^{-9}, and the number 7,390,000,000,000 is written 7.39×10^{12}.

exponential function — A function with the variable as an exponent that has the form $y = abx$, where $b > 0$ and $b \neq 1$.

exponential growth — An exponential function that has the form $y = ab^x$, where $a > 0$ and $b > 1$.

exponential decay — An exponential function that has the form $y = ab^x$, where $a > 0$ and $0 < b < 1$.

Pre-viewing Discussion

- Give students graphs of a linear function and an exponential function, but do not label them as such. Have students make a table of values based on the graph of each function, and then use the information in the table to describe differences between the two graphs.
- Discuss how to plot ordered pairs in the first quadrant of a coordinate grid and how to read information from a coordinate graph.

Problems

1. Simplify each expression. Assume x and y are not equal to zero.

   (a) $\dfrac{2x^5y^3}{xy^4}$ (b) $\dfrac{20x(y^2)^3}{(2x)^2y^4}$ (c) $\dfrac{(35xy^{-2})^0}{(5x^3y)^0}$

2. Consider the numbers 0.00000000045 and 93,000,000.

   (a) When written in scientific notation, would you expect either of these numbers to have a power of ten with a negative exponent? Why or why not?

   (b) Express each number in scientific notation.

3. The country of Exponentia has a growth rate that can be expressed by the exponential function $y = C(1.055)^t$, where C is the current population and t is the time in years from the present. If the current population is 200,000 people, determine the population of Exponentia in ten years. Round your answer to the nearest thousand.

(Continued)

4. Daniel buys a new car for $40,000. The depreciation of the car can be expressed by the exponential function $y = C(0.88)^t$, where C is the cost of the new car and t is the age of the car in years. Find the value of the car after five years. Round your answer to the nearest cent.

Solutions

1. Following the order of operations, all parentheses are removed first, after which exponents are simplified, and finally multiplication and division is performed:

   (a) $\dfrac{2x^5y^3}{xy^4} = \dfrac{2x^5}{xy^4y^3} = \dfrac{2x^4}{y^7}$

   (b) $\dfrac{20x(y^2)^3}{(2x)^2y^4} = \dfrac{20xy^6}{4x^2y^4} = \dfrac{5y^2}{x}$

   (c) $\dfrac{(35xy^{-2})^0}{(5x^3y)^0} = \dfrac{1}{1} = 1$ Remember that any nonzero number raised to the zero power equals 1.

2. (a) Since $10^0 = 1$, numbers less than 1, like 0.00000000045, will have a power of ten with a negative exponent when written in scientific notation.

   (b) $0.00000000045 = 4.5 \times 10^{-10}$

   $93{,}000{,}000 = 9.3 \times 10^7$

3. To find the population of Exponentia in 10 years, substitute the current population ($C = 200{,}000$) and the time in years from the present ($t = 10$) into the given exponential growth function and simplify:

   $y = C(1.055)^t$
   $y = 200{,}000(1.055)^{10}$
   $y \approx 200{,}000(1.708144)$
   $y \approx 341{,}628.8$

   The population of Exponentia will be approximately 342,000.

4. To find the value of the car after 5 years, substitute the purchase price of the new car ($C = 40{,}000$) and the time in years from when the car was purchased ($t = 5$) into the given exponential decay function and simplify:

   $y = C(0.88)^t$
   $y = 40{,}000(0.88)^5$
   $y \approx 40{,}000(0.527732)$
   $y \approx 21{,}109.28$

   The value of the car will be approximately $21,109.28.